# GRAPH THEORY 

## Tutorial - 6

1. Let $\mathbf{G}$ be a simple n-vertex graph. Prove that, $\mathbf{G}$ is $\mathbf{k}$-connected for all $k$ with $k \leq \mathbf{2 \delta}(\mathbf{G})+\mathbf{2 - n}$.
2. Let $G$ be a simple n-vertex graph.
(a) Prove that, if $\delta(G) \geq\lfloor n / 2\rfloor$, then $\kappa^{\prime}(\mathrm{G})=\boldsymbol{\delta}(\mathrm{G})$.
(b) If $d(x)+d(y) \geq n-1$ whenever $x \nleftarrow y$, then prove that $\kappa^{\prime}(G)=\delta(G)$.
3. Let G be a simple graph with diameter 2 and let $[\mathrm{S}, \overline{\mathrm{S}}$ ] be a minimum edge cut with $|\mathbf{S}| \leq|\bar{S}|$.
(a) Prove that, every vertex of S has a neighbor in $\overline{\mathbf{S}}$.
(b) Use part-(a) and prove that, $\kappa^{\prime}(G)=\delta(G)$.
4. Prove that the symmetric difference of two different edge cuts is an edge cut.
